OCR
Oxford Cambridge and RSA

Thursday 19 May 2022 – Afternoon

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + ... + {}^nC_r a^{n-r}b^r + ... + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ... + \frac{n(n-1)...(n-r+1)}{r!}x^r + ... \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

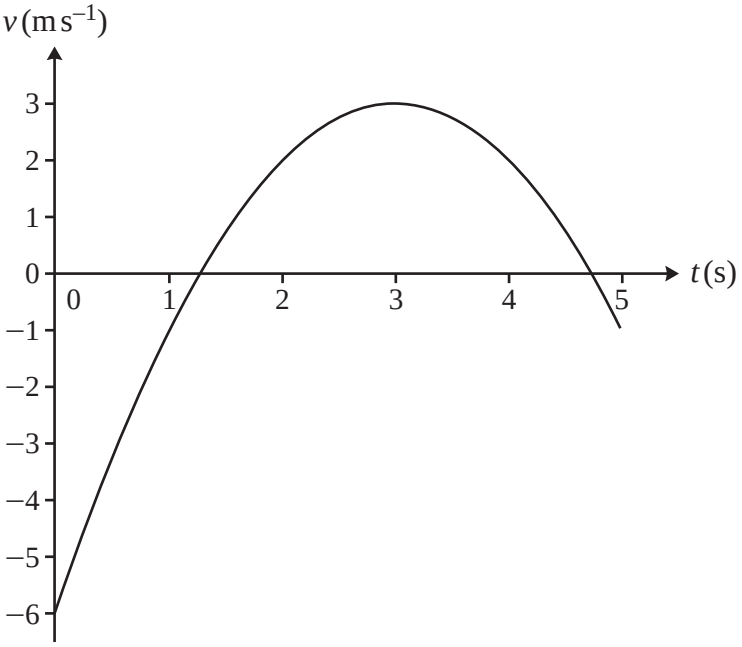
Answer **all** the questions.

1 Rationalise the denominator of the fraction $\frac{2+\sqrt{n}}{3+\sqrt{n}}$, where n is a positive integer. **[3]**

2 **(a)** Determine the value of $\frac{100!}{98!}$. **[2]**

(b) Find the coefficient of x^{98} in the expansion of $(1+x)^{100}$. **[1]**

3 The velocity-time graph for the motion of a particle is shown below. The velocity $v\,\text{m s}^{-1}$ at time $t\,\text{s}$ is given by $v = -t^2 + 6t - 6$ where $0 \leqslant t \leqslant 5$.



(a) Find the times at which the velocity is $2\,\text{m s}^{-1}$. **[2]**

(b) Write down the greatest speed of the particle. **[1]**

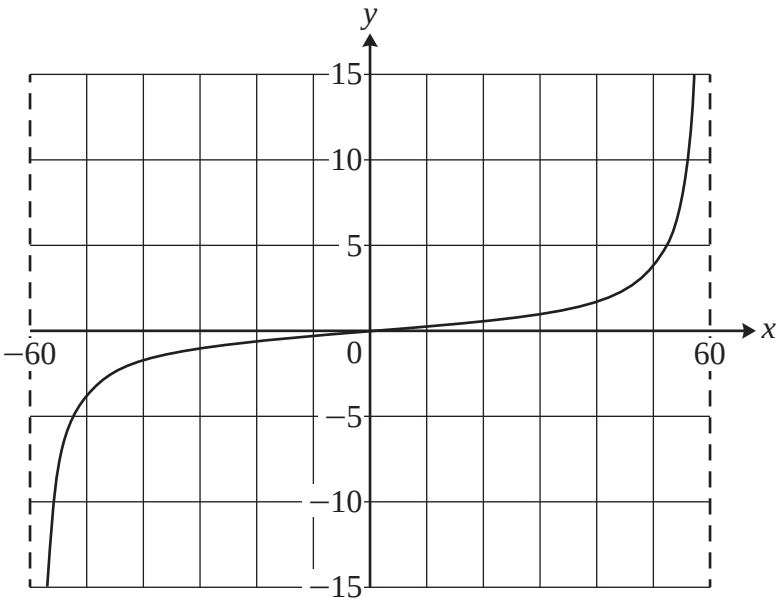
4 The quadratic function $f(x)$ is given by $f(x) = x^2 - 3x + 2$.

(a) Write $f(x)$ in the form $(x+a)^2 + b$, where a and b are constants. **[2]**

(b) Write down the coordinates of the minimum point on the graph of $y = f(x)$. **[2]**

(c) Describe fully the transformation that maps the graph of $y = f(x)$ onto the graph of $y = (x+1)^2 - \frac{1}{4}$. **[2]**

5 Part of the graph of $y = f(x)$ is shown below. The graph is the image of $y = \tan x°$ after a stretch in the x-direction.



(a) Find the equation of the graph. **[2]**

(b) Write down the period of the function $f(x)$. **[1]**

(c) **In this question you must show detailed reasoning.**

Find all the roots of the equation $f(x) = 1$ for $0° \leqslant x° \leqslant 360°$. **[3]**

6 The gradient of a curve is given by the equation $\frac{dy}{dx} = 6x^2 - 20x + 6$. The curve passes through the point (2, 6).

(a) Find the equation of the curve. **[3]**

(b) Verify that the equation of the curve can be written as $y = 2(x+1)(x-3)^2$. **[2]**

(c) Sketch the curve, indicating the points where the curve meets the axes. **[3]**

7 In this question the unit vectors **i** and **j** are directed east and north respectively.

A canal narrowboat of mass 9 tonnes is pulled by two ropes. The tensions in the ropes are $(450\mathbf{i} + 20\mathbf{j})$ N and $(420\mathbf{i} - 20\mathbf{j})$ N. The boat experiences a resistance to motion **R** of magnitude 300 N.

(a) Explain what it means to model the boat as a particle. **[1]**

The boat is travelling in a straight line due east.

(b) Find the equation of motion of the boat. **[2]**

(c) Find the acceleration of the boat giving your answer as a vector. **[1]**

8 A team of volunteers donates cakes for sale at a charity stall. The number of cakes that can be sold depends on the price. A model for this is $y = 190 - 70x$, where y cakes can be sold when the price of a cake is £x.

(a) Find how many cakes could be given away for free according to this model. **[1]**

The number of volunteers who are willing to donate cakes goes up as the price goes up. If the cakes sell for £1.20 they will donate 50 cakes, but if they sell for £2.40 they will donate 140 cakes. They use the linear model $y = mx + c$ to relate the number of cakes donated, y, to the price of a cake, £x.

(b) Find the values of the constants m and c for which this linear model fits the two data points. **[2]**

(c) Explain why the model is **not** suitable for very low prices. **[1]**

(d) The team would like to sell all the cakes that they donate.

Find the set of possible prices that the cakes could have to achieve this. **[3]**

9 A tractor of mass 1800 kg uses a towbar to pull a trailer of mass 1000 kg on a level field. The tractor and trailer experience resistances to motion of 1600 N and 800 N respectively. The tractor provides a driving force of 6600 N.

(a) Draw a force diagram showing all the horizontal forces acting on the tractor and trailer. **[2]**

(b) Find the tension in the towbar. **[4]**

10 A triangle has vertices A (1, 4), B (7, 0) and C $(-4, -1)$.

(a) Show that the equation of the line AC is $y = x + 3$. **[2]**

M is the midpoint of AB. The line AC intersects the x-axis at D.

(b) Determine the angle DMA. **[7]**

11 A sports car accelerates along a straight road from rest. After 5 s its velocity is $9\,\text{m}\,\text{s}^{-1}$.

In model A, the acceleration is assumed to be constant.

(a) Calculate the distance travelled by the car in the first 5 seconds according to model A. **[2]**

In model B, the velocity v in $\text{m}\,\text{s}^{-1}$ is given by $v = 0.05t^3 + kt$, where t is the time in seconds after the start and k is a constant.

(b) Find the value of k which gives the correct value of v when $t = 5$. **[2]**

(c) Using this value of k in model B, calculate the acceleration of the car when $t = 5$. **[2]**

The car travels 16 m in the first 5 seconds.

(d) Show that model B, with the value of k found in part **(b)**, better fits this information than model A does. **[3]**

12 Below is a faulty argument that appears to show that the gradient of the curve $y = x^2$ at the point (3, 9) is 1.

> Consider the chord joining (3, 9) to the point $(3+h, (3+h)^2)$
>
> The gradient is $\frac{(3+h)^2-9}{h} = \frac{6h+h^2}{h}$
>
> When $h = 0$ the gradient is $\frac{0}{0}$ so the gradient of the curve is 1

(a) Identify a fault in the argument. **[1]**

(b) Write a valid first principles argument leading to the correct value for the gradient at (3, 9). **[3]**

(c) Find the equation of the normal to the curve at the point (3, 9). **[2]**

END OF QUESTION PAPER